Problems: Week2

2-1. A pendulum on Earth ($g_E = 9.8 m/s^2$) has a period of 1 sec. What is its length? If you take it to the moon what will its period be there if $g_M = \frac{g_E}{6}$? If you want the period to be 1 sec on the moon by what factor must you change the length?

2-2. A spring-mass system is oscillating on a horizontal frictionless surface and its position is given by $x = 0.05\ m\ \cos\omega t$. For what values of x will (i) its kinetic energy be maximum, (ii) its potential energy be maximum (iii) its kinetic energy equal its potential energy.

2-3. If the system of problem S-2 is hung from a ceiling, what will be the change in its frequency? Why?

2-4. A wave on a stretched string is represented by

$\underset{\to}{y} = 0.01\sin(6.28x - 12.56t)\hat{y}$

where distances are in meters and times in seconds.

(i) Is this longitudinal or transverse? (ii) What is its wavelength, frequency, velocity?

2-5. A sinusoidal wave of amplitude A and frequency ω, travelling on a stretched string carries

$$P = \frac{1}{2}A^2\omega^2\frac{T}{v}, \text{ where } v = \sqrt{\frac{T}{\mu}}$$

Joules of energy per second. Here T is the tension and v, the wave speed. Changing only one factor at a time how would P change if you

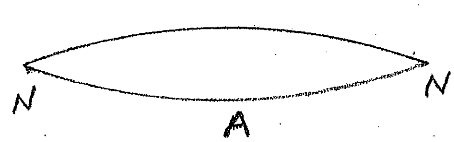
(i) double A (ii) halve ω or (iii) increase T by a factor of 3.

2-6. A sine wave $y_i = A_i\sin(kx + \omega t)$ is launched on one string with velocity $\underset{\to}{v} = \frac{-\omega}{k}\hat{x}$. At x = 0, it encounters a second string where velocity is v' and gives rise to a reflected wave $y_r = A_r\sin(kx - \omega t)$ and a transmitted wave $y_t = A_t\sin(k'x + \omega' t)$. (i) What is the relation between ω' and ω? Why? (ii) What determines k'? (iii) Given that $\frac{A_r}{A_i} = \frac{v - v'}{v + v'}$ and $\frac{A_t}{A_i} = \frac{2v'}{v + v'}$ show that during reflection there is a phase change of Π if $v' << v$.

2-7. Using the result of problem 2-6 and the superposition principle, show that where the incident and reflected waves combine, nodes appear at $x = 0, n\frac{\lambda}{2} (n = 1,2,...)$ and antinodes occur at $x = (2n+1)\frac{\lambda}{4} (n = 1,2,...)$.

2-8. A wire of length 1m and mass 0.001kg has a tension of 10^3N in it and is fixed at both ends. Calculate the frequencies of the 3rd and 5th harmonic modes.

2-9. In the standing wave experiment that you have performed (a) why does the lowest (n = 1) mode require the largest tension (hanging mass)? (b) by what factor must you change the mass to get the n = 3 mode?



2-10. The intensity of a periodic sound wave of amplitude s_m and frequency ω in a gas at pressure P_o is

$$I = \frac{1}{2} s_m^2 \omega^2 \frac{\gamma P_o}{v_s}$$

where $\gamma = \frac{C_P}{C_v}$ and v_s is the speed of sound. Calculate s_m for air if P_o=$10^5 N/m^2$, $\gamma = 1.4$, v_s= 330m/s and I= I_o= 10^{-12} $Watt/m^2$; the quietest sound that can be heard.

2-11. A periodic sound wave can be thought of as a displacement wave

$$s = s_m \sin(kx - \omega t)$$

or a pressure wave

$$P = P_o - \gamma P_o s_m \cos(kx - \omega t)$$

Why is the pressure wave always $\frac{\pi}{2}$ out of phase with the displacement, that is, why is pressure variation maximum where displacement is zero and vice versa?

2-12. In air the speed of sound at room temperature is about $330m/s$. What must be the wavelengths of mechanical waves to be called sound?

2-13. The speed of sound in a gas is written as

$$V_s = \sqrt{\frac{\gamma k_B T}{m}} \text{ where } \gamma = \frac{C_p}{C_V}$$

Why is there a "γ" in this equation?

2-14. Using the formula of 2-13 compare the speeds of sound in Helium $[\gamma = \frac{5}{3}, m = 4mp]$ and air $[\gamma = \frac{7}{5}, m \cong 30mp]$.

2-15. Compare the speed of sound in He ($\gamma = \frac{5}{3}$) with the r.m.s. speed $v_{rms} = \sqrt{\frac{3k_B T}{m}}$ of He atoms.